

EFFECTIVE ACTION AND ELECTROMAGNETIC POLARIZABILITIES OF NUCLEONS IN QCD STRING THEORY

S.I. Kruglov ¹

*University of Toronto at Scarborough,
Physical and Environmental Sciences Department,
1265 Military Trail, Toronto, Ontario, Canada M1C 1A4*

Abstract

The effective action for baryons in the external electromagnetic fields is obtained on the basis of the QCD string theory. The area law of the Wilson loop integral, the approximation of the Nambu-Goto straight-line strings, and the asymmetric quark-diquark structure of nucleons are used to simplify the problem. The spin-orbit and spin-spin interactions of quarks are treated as a perturbation. With the help of the virial theorem we estimate the mean radii of nucleons in terms of the string tension and the Airy function zeros. On the basis of the perturbation theory in small external electromagnetic fields we derive the electromagnetic polarizabilities of nucleons. The electric and diamagnetic polarizabilities of a proton are $\bar{\alpha}_p = 10 \times 10^{-4} fm^3$, $\beta_p^{dia} = -8 \times 10^{-4} fm^3$ and for a neutron we find $\bar{\alpha}_n = 4.2 \times 10^{-4} fm^3$, $\beta_n^{dia} = -5.4 \times 10^{-4} fm^3$. Reasonable values of the magnetic polarizabilities $\beta_p = (5 \pm 3) \times 10^{-4} fm^3$, $\beta_n = (7.6 \pm 3) \times 10^{-4} fm^3$ are estimated using the Δ contribution to the paramagnetic polarizability of the nucleons.

1 Introduction

In recent papers [1,2] we have calculated some electromagnetic characteristics of mesons in the framework of the QCD string theory [3,4]. Starting with QCD and excepting the area law for the Wilson loop integral, the effective action for mesons in external electromagnetic fields was derived. This approach takes into account the chiral symmetry breaking and the confinement of quarks [3,4]. This work is the continuation of [5] on the case of baryons,

¹E-mail: krouglov@utsc.utoronto.ca

i.e. the three-quark system. We make some assumptions such as (i) the area law behavior of the Wilson loop, (ii) the straight-line approximation of the strings, (iii) the handling of spin degrees of freedom as a perturbation, (iiii) the quark-diquark structure of baryons. The spin-orbit, spin-spin interactions and Coulomb like short-range contributions are also neglected, as a first approximation. The area-law of the Wilson loop integral (or the confinement of quarks) was confirmed by the Monte-Carlo simulations and by the theoretical considerations. One of the approaches which explains the confinement of quarks is the approach [6] introducing stochastic gluon vacuum fields. The second one uses the dual Meissner effect to guarantee the confinement of colour [7,8]. In our consideration, the confinement of quarks is postulated by excepting the area-law of the Wilson loop. The use of the straight-line approximation for the Nambu-Goto string is useful to simplify the calculations. Neglecting short-range interactions is justified by considering large distances. The quark-diquark structure of baryons was considered in the framework of the nonrelativistic [9] and relativistic [10] models. In this case the linear baryon Regge trajectories have the same slope as for mesons [3].

The paper is organized as follows. In Section 2 we describe the general background and derive the Green function of a baryon. The effective action for baryons in external electromagnetic fields based on the proper time method and Feynman path-integrals is found in Section 3. Section 4 contains the calculation of average distances between quarks and electric polarizabilities of nucleons. We derive diamagnetic polarizabilities of protons and neutrons using the perturbative expansion in the small magnetic fields in Section 5. Section 6 contains a conclusion.

2 The Green function of three-quark system

Here we derive the Green function of three-quark system using the Schwinger proper time method and the Feynman path-integrals. Our goal is to calculate some electromagnetic characteristics of nucleons. For this purpose we need the effective action for baryons in external electromagnetic fields. The method of the Green functions will be explored. Let us consider the Lorentz-covariant and gauge invariant combination of three-quark, colourless system (baryon) [4]

$$X_B(x, y, z, C_i) = \epsilon_{abc} [\Phi(Z_0, x)q(x)]_a [\Phi(Z_0, y)q(y)]_b [\Phi(Z_0, z)q(z)]_c, \quad (1)$$

where $q(x)$ is a quark bispinor; a, b, c are colour indexes so that $[\Phi(Z_0, x)q(x)]_a = \Phi_{aa'}(Z_0, x)q_{a'}(x)$ and ϵ_{abc} is the Levi-Civita symbol ($\epsilon_{123} = 1$). We imply that quark fields $q(x)$, $q(y)$ and $q(z)$ possess the definite flavours which will be specified later at the considering nucleons. As usual there is a summation on repeating indexes. The gauge invariance is guaranteed here by introducing the parallel transporter [4]:

$$\Phi(Z_0, x) = P \exp \left\{ ig \int_x^{Z_0} A_\mu dz_\mu \right\}, \quad (2)$$

where P is the ordering operator along the contour C_1 of integration, g is the coupling constant, $A_\mu = A_\mu^a \lambda^a$; A_μ^a are the gluonic fields; λ^a are the Gell-Mann matrices and Z_0 is the string junction. Although we have an attractive feature - gauge invariance, the function (1) depends on the form of the contour C . As $X_B(x, y, z, C_i)$ is a gauge invariant object, it obeys the Gauss law on the spacelike surface Σ . Three quarks (with fields $q(x)$, $q(y)$ and $q(z)$) are situated in the four-points x, y, z , respectively and connected with the four-point Z_0 which is arbitrary four-point. Later, the position of Z_0 will be defined by requiring to have the minimal area for the world surface of three-quark system [4]. The path of integration in Eq. (2) is also arbitrary.

Let the four-points x, y, z and x', y', z' be the initial and final positions of three quarks, respectively. Two particle quantum Green function is defined as [4]:

$$G(xyz, x'y'z') = \langle X_B(x, y, z, C_i) \bar{X}_B(x', y', z', C'_i) \rangle, \quad (3)$$

where $\bar{X}_B(x', y', z', C'_i)$ corresponds to the final state of a baryon:

$$\bar{X}_B(x', y', z', C'_i) = \epsilon_{mnk} [\bar{q}(x')\Phi(x', Z'_0)]_m [\bar{q}(y')\Phi(y', Z'_0)]_n [\bar{q}(z')\Phi(z', Z'_0)]_k. \quad (4)$$

Here $\bar{q} = q^+ \gamma_4$; q^+ is the Hermitean conjugate quark field; γ_μ are the Dirac matrices and the brackets $\langle \dots \rangle$ mean the path-integrating over gluonic and quark fields:

$$\langle X_B \bar{X}_B \rangle = \int D\bar{q} Dq DA_\mu \exp \{ i S_{QCD} \} X_B \bar{X}_B, \quad (5)$$

with the QCD action S_{QCD} . We imply that the measure DA_μ in the path integral (5) includes the well known weight for gluonic fields [11]. The Minkowski space is used here but it is not difficult to go into Euclidean space to have well defined path-integrals.

Let us introduce the generating functional for Green's function to calculate the path-integral (5) with respect to quark fields:

$$Z[\bar{\eta}, \eta] = \int D\bar{q} Dq \exp \left\{ iS_{QCD} + i \int dx (\bar{q}_a(x) \eta_a(x) + \bar{\eta}_a(x) q_a(x)) \right\}, \quad (6)$$

where we introduce the external colour anticommutative sources $\eta_a, \bar{\eta}_a$. Then the Green function (3) can be written as

$$\begin{aligned} G(xyz, x'y'z') &= \int DA_\mu \left[\epsilon_{abc} \epsilon_{mnk} \Phi_{aa'}(Z_0, x) \Phi_{bb'}(Z_0, y) \Phi_{cc'}(Z_0, z) \right. \\ &\quad \times \Phi_{m'm}(x', Z'_0) \Phi_{n'n}(y', Z'_0) \Phi_{k'k}(z', Z'_0) \\ &\quad \left. \times \delta^6 / \delta \bar{\eta}_{a'}(x) \delta \bar{\eta}_{b'}(y) \delta \bar{\eta}_{c'}(z) \delta \eta_{m'}(x') \delta \eta_{n'}(y') \delta \eta_{k'}(z') Z[\bar{\eta}, \eta] \right]_{\eta=\bar{\eta}=0}. \end{aligned} \quad (7)$$

In Eq. (7) Z_0 and Z'_0 are the initial and final positions of the string junction, respectively. The total surface which consists of world motions of quarks and the path of the string junction must be minimal. This requirement defines the path of the string junction [4]. Now it is possible to integrate the path-integral in Eq. (7) over quark fields \bar{q}, q as expression (6) is a Gaussian integral. We may represent the QCD action in the form

$$S_{QCD} = S(A) - \int dx \bar{q}(x) (\gamma_\mu D_\mu + m) q(x), \quad (8)$$

where $S(A)$ is an action for gluonic fields with the included ghost fields, $D_\mu = \partial_\mu - igA_\mu$; m is the quark mass matrix and we imply the summation on colour and flavour indexes. Inserting Eq. (8) into Eq. (6) and integrating with respect to quark fields, we arrive at the expression

$$Z[\bar{\eta}, \eta] = \det(-\gamma_\mu D_\mu - m) \exp \left\{ iS(A) + i \int dx dy \bar{\eta}(x) S(x, y) \eta(y) \right\}, \quad (9)$$

where the classical quark Green function $S(x, y)$ is the solution of the equation

$$(\gamma_\mu D_\mu + m) S(x, y) = \delta(x - y). \quad (10)$$

Using Eq. (9) and calculating the variation derivatives in Eq. (7) we find the quantum Green function of a baryon:

$$G(xyz, x'y'z') = \int DA_\mu \det(-\gamma_\mu D_\mu - m) \exp \{ iS(A) \} \epsilon_{abc} \epsilon_{mnk}$$

$$\begin{aligned}
& \times \left[S_{bm}^\Phi(y, x') S_{an}^\Phi(x, y') S_{ck}^\Phi(z, z') - S_{am}^\Phi(x, x') S_{bn}^\Phi(y, y') S_{ck}^\Phi(z, z') \right. \\
& + S_{cm}^\Phi(z, x') S_{bn}^\Phi(y, y') S_{ak}^\Phi(x, z') - S_{cm}^\Phi(z, x') S_{an}^\Phi(x, y') S_{bk}^\Phi(y, z') \\
& \left. + S_{am}^\Phi(x, x') S_{cn}^\Phi(z, y') S_{bk}^\Phi(y, z') - S_{bm}^\Phi(y, x') S_{cn}^\Phi(z, y') S_{ak}^\Phi(x, z') \right],
\end{aligned} \tag{11}$$

where we introduce the following notation for the covariant Green function

$$S_{am}^\Phi(x, x') = \Phi_{aa'}(Z_0, x) S_{a'm'}(x, x') \Phi_{m'm}(x', Z'_0). \tag{12}$$

The functional determinant in Eq. (9) describes the contribution from the vacuum polarization and gives the addition quark loops. As a first approximation we neglect the contribution of the loops. The presence of different terms in Eq. (11) is connected with the permutations of quark fields because the quantum Green function being considered. As different terms in Eq. (11) have the same structure, we consider in detail only one term. Neglecting the functional determinant we find the approximate expression for the baryon Green function

$$G_1(xyz, x'y'z') = - \int DA_\mu \exp \{iS(A)\} \epsilon_{abc} \epsilon_{mnk} S_{am}^\Phi(x, x') S_{bn}^\Phi(y, y') S_{ck}^\Phi(z, z'). \tag{13}$$

Expression (13) is the basic formula for deriving effective action for baryons.

3 Effective action for baryons

To calculate some electromagnetic characteristics of nucleons we consider baryons in external electromagnetic fields. Then covariant derivatives become $D_\mu = \partial_\mu - igA_\mu - iQA_\mu^{el}$, where A_μ^{el} is the vector-potential of electromagnetic fields, Q is the charge matrix of quarks, $Q = \text{diag}(e_1, e_2, \dots, e_{N_f})$, e_i are charges of quarks, N_f is the number of flavours. Using the proper time method and Feynman path-integrals we found in [2] the Green function of a quark which is the solution to Eq. (10):

$$\begin{aligned}
& S(x, x') = i \int_0^\infty ds \int_{z(0)=x'}^{z(s)=x} Dz \left(m - \frac{i}{2} \gamma_\mu \dot{z}_\mu(t) \right) \\
& \times P_\Sigma \exp \left\{ i \int_0^s dt \left[\frac{1}{4} \dot{z}_\mu^2(t) - m^2 + e \dot{z}_\mu(t) A_\mu^{el}(z) + \Sigma_{\mu\nu} (e F_{\mu\nu}^{el} + g F_{\mu\nu}) \right] \right\} \Phi(x, x'),
\end{aligned} \tag{14}$$

where $F_{\mu\nu}^{el} = \partial_\mu A_\nu^{el} - \partial_\nu A_\mu^{el}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu]$ are the strength tensors of electromagnetic and gluonic fields, respectively; $\Sigma_{\mu\nu} = -(i/4) [\gamma_\mu, \gamma_\nu]$ are the spin matrices; P_Σ is the ordering operator of the spin matrices $\Sigma_{\mu\nu}$; $\dot{z}_\mu(t) = \partial z_\mu(t)/\partial t$; $z_\mu(t)$ is the path of the quark with the boundary conditions $z_\mu(0) = x'_\mu$, $z_\mu(s) = x_\mu$ and $\Phi(x, x')$ is the path ordered product (see Eq. (2)). Inserting Eq. (14) into Eq. (12) we find

$$S_{am}^\Phi(x, x') = -i \int_0^\infty ds \int_{z(0)=x'}^{z(s)=x} Dz \left(m - \frac{i}{2} \gamma_\mu \dot{z}_\mu(t) \right) P_\Sigma \times \exp \left\{ i \int_0^s dt \left[\frac{1}{4} \dot{z}_\mu^2(t) - m^2 + e \dot{z}_\mu(t) A_\mu^{el}(z) + \Sigma_{\mu\nu} (e F_{\mu\nu}^{el} + g F_{\mu\nu}) \right] \right\} (\Phi_{C_x}(Z_0, Z'_0))_{am}, \quad (15)$$

where the contour C_x in Eq. (15) consists of lines between Z_0 , x and Z'_0 , x' and path $z_\mu(t)$. Using the expression (15) for each quark, the baryon Green function (13) becomes

$$G_1(xyz, x'y'z') = i \int_0^\infty \prod_j ds_j \int \prod_j Dz^{(j)} \left(m_j - \frac{i}{2} \gamma_\mu \dot{z}_\mu^{(j)}(t_j) \right) \times P_\Sigma \prod_j \exp \left\{ \int_0^{s_j} dt_j \Sigma_{\mu\nu} \frac{\delta}{\delta \sigma_{\mu\nu}^{(j)}(t_j)} \right\} \times \exp \left\{ i \sum_j \int_0^{s_j} dt_j \left[\frac{1}{4} \left(\dot{z}_\mu^{(j)}(t_j) \right)^2 - m_j^2 + e_j \dot{z}_\mu^{(j)}(t_j) A_\mu^{el}(z^{(j)}) + e_j \Sigma_{\mu\nu} F_{\mu\nu}^{el} \right] \right\} \times \langle W(C_x C_y C_z) \rangle, \quad (16)$$

where $j = 1, 2, 3$; e_j is the charge of the j -th quark; the boundary conditions $z_\mu^{(1)}(0) = x'_\mu$, $z_\mu^{(1)}(s_1) = x_\mu$, $z_\mu^{(2)}(0) = y'_\mu$, $z_\mu^{(2)}(s_2) = y_\mu$, $z_\mu^{(3)}(0) = z'_\mu$, $z_\mu^{(3)}(s_3) = z_\mu$ are used here and the Wilson loop is given by (see [4])

$$\langle W(C_x C_y C_z) \rangle = \epsilon_{abc} \epsilon_{mnk} \langle (\Phi_{C_x}(Z_0, Z'_0))_{am} (\Phi_{C_y}(Z_0, Z'_0))_{bn} (\Phi_{C_z}(Z_0, Z'_0))_{ck} \rangle. \quad (17)$$

We took into account that [4]

$$\exp \left\{ ig \int_0^s dt \Sigma_{\mu\nu} F_{\mu\nu} \right\} W(C) = \exp \left\{ \int_0^s dt \Sigma_{\mu\nu} \frac{\delta}{\delta \sigma_{\mu\nu}(t)} \right\} W(C),$$

where $\sigma_{\mu\nu}(t)$ is the surface around the point $z_\mu(t)$.

Contours C_x, C_y, C_z correspond to three quarks which have paths $z_\mu^{(1)}, z_\mu^{(2)}, z_\mu^{(3)}$ and masses m_1, m_2, m_3 , respectively. Relationship (16) is the generalization of one [3,4] for the case of quarks placed in external electromagnetic fields which possess spins.

We imply further that the average distance between quarks $\langle r \rangle$ is greater than the time fluctuations (in units $c = \hbar = 1$) of the gluonic fields T_g : $\langle r \rangle > T_g$. As $T_g \simeq 0.2 \div 0.3$ fm [12,13] so the condition $\langle r \rangle > T_g$ is valid not only for asymptotic baryon states of the Regge trajectories with large angular momenta of the baryon but also for lower baryon states. The asymptotic of the average Wilson loop integral obeys then the area law and is given by (in the Minkowski space):

$$\langle W(C_x C_y C_z) \rangle = \exp \{ -i\sigma (S_1 + S_2 + S_3) \}, \quad (18)$$

where σ is the string tension and S_j ($j = 1, 2, 3$) is the minimal surface bounded by the trajectories of the quark q_j and string junction Z_0 . The path of the string junction is defined by the requirement that the sum $S_1 + S_2 + S_3$ is minimal [3,4]. Following [3,4], new variables are introduced:

$$\tau = \frac{t_1 T}{s_1} = \frac{t_2 T}{s_2} = \frac{t_3 T}{s_3}, \quad \mu_j = \frac{T}{2s_j}, \quad (19)$$

where τ means the proper time for every quark and μ_j ($j = 1, 2, 3$) is the dynamical mass of the j -th quark. Using Eqs. (18), (19) from Eq. (16) we arrive at

$$\begin{aligned} G_1(xyz, x'y'z') = & -i \frac{T^3}{8} \int_0^\infty \prod_j \frac{d\mu_j}{\mu_j^2} \int \prod_j D z^{(j)} \left(m_j - i\mu_j \gamma_\mu \dot{z}_\mu^{(j)}(\tau) \right) \times \\ & \times P_\Sigma \prod_j \exp \left\{ \frac{1}{2\mu_j} \Sigma_{\mu\nu} \int_0^T d\tau \frac{\delta}{\delta \sigma_{\mu\nu}^{(j)}(\tau)} \right\} \times \\ & \times \exp \left\{ i \int_0^T d\tau \sum_j \left[\frac{1}{2} \mu_j \left(\dot{z}_\mu^{(j)}(\tau) \right)^2 - \frac{m_j^2}{2\mu_j} + e_j \dot{z}_\mu^{(j)}(\tau) A_\mu^{el}(z^{(j)}) + \frac{e_j}{2\mu_j} \Sigma_{\mu\nu} F_{\mu\nu}^{el} \right] \right. \\ & \left. - i\sigma (S_1 + S_2 + S_3) \right\}. \end{aligned} \quad (20)$$

The integral in the last exponential factor of Eq. (20) represents the effective action for three-quark system (baryon) taking into account the spins of quarks. So T is the time of the observation, τ is the proper time of

quarks; m_j and μ_j are the current and dynamical masses of j -th quark. It follows from Eq. (20) that there is integration over dynamical masses μ_j to get the Green function of the three-quark system. Pre-exponential factors in Eq. (20) allow us to calculate in principle the spin-spin and spin-orbit contributions to the effective action. As a first approximation (see [3,4]) we neglect the short-range spin corrections and consider therefore scalar quarks. The terms $e_j \Sigma_{\mu\nu} F_{\mu\nu}^{el}$ which describe the interaction of the magnetic field with the spins of quarks will be omitted. With this assumption we arrive at the effective action for baryons in external electromagnetic fields

$$B = \int_0^T d\tau \sum_j \left[\frac{1}{2} \mu_j \left(\dot{z}_\mu^{(j)}(\tau) \right)^2 - \frac{m_j^2}{2\mu_j} + e_j \dot{z}_\mu^{(j)}(\tau) A_\mu^{el}(z^{(j)}) \right] - \sigma (S_1 + S_2 + S_3). \quad (21)$$

The case when electromagnetic fields are absent was considered in [3,4]. The terms which describe the interaction of quarks with electromagnetic fields are essential for us because we are going to calculate electromagnetic characteristics of nucleons. It is convenient to introduce new variables R_μ , ξ_μ and η_μ instead of $z_\mu^{(j)}$ in accordance with relationships [3,4]:

$$\begin{aligned} z_\mu^{(1)} &= R_\mu + \left(\frac{\mu\mu_3}{M(\mu_1 + \mu_2)} \right)^{1/2} \xi_\mu - \left(\frac{\mu\mu_2}{\mu_1(\mu_1 + \mu_2)} \right)^{1/2} \eta_\mu, \\ z_\mu^{(2)} &= R_\mu + \left(\frac{\mu\mu_3}{M(\mu_1 + \mu_2)} \right)^{1/2} \xi_\mu + \left(\frac{\mu\mu_1}{\mu_2(\mu_1 + \mu_2)} \right)^{1/2} \eta_\mu, \\ z_\mu^{(3)} &= R_\mu - \left(\frac{\mu(\mu_1 + \mu_2)}{M\mu_3} \right)^{1/2} \xi_\mu, \end{aligned} \quad (22)$$

where R_μ is the center of mass coordinate of a baryon; ξ_μ and η_μ are relative coordinates of quarks, $M = \mu_1 + \mu_2 + \mu_3$ is the sum of dynamical masses of quarks. The arbitrary mass parameter μ in Eq. (22) defines the scale of relative coordinates ξ_μ and η_μ . After the substitution (22), the measure $\prod_j Dz^{(j)}$ transforms into $DRD\eta D\xi$ in path-integral (20).

Let us consider the uniform and constant external electromagnetic fields. Then the vector-potential of electromagnetic fields can be represented as

$$A_v^{el}(z^{(j)}) = \frac{1}{2} F_{\mu\nu}^{el} z_\mu^{(j)}. \quad (23)$$

Inserting Eqs. (22), (23) into Eq. (21) we find the effective action for the three quark system (see [5]) in the form

$$B = \int_0^T d\tau \left[\frac{M}{2} \dot{R}_\nu^2 + \frac{\mu}{2} (\dot{\xi}_\nu^2 + \dot{\eta}_\nu^2) - \sum_j \frac{m_j^2}{2\mu_j} \right] - \sigma (S_1 + S_2 + S_3) + \Delta B, \quad (24)$$

$$\begin{aligned} \Delta B = & \frac{1}{2} F_{\nu\mu}^{el} \int_0^T d\tau \left[e \dot{R}_\mu R_\nu + \lambda \dot{\xi}_\mu \xi_\nu + \rho \dot{\eta}_\mu \eta_\nu \right. \\ & \left. + \gamma (\dot{R}_\mu \xi_\nu + \dot{\xi}_\mu R_\nu) + \delta (\dot{R}_\mu \eta_\nu + \dot{\eta}_\mu R_\nu) + \delta \sqrt{\frac{\mu\mu_3}{(\mu_1 + \mu_2)M}} (\dot{\xi}_\mu \eta_\nu + \dot{\eta}_\mu \xi_\nu) \right], \end{aligned} \quad (25)$$

where we introduce parameters:

$$\begin{aligned} \gamma &= \sqrt{\frac{\mu}{M}} \left[(e_1 + e_2) \sqrt{\frac{\mu_3}{\mu_1 + \mu_2}} - e_3 \sqrt{\frac{\mu_1 + \mu_2}{\mu_3}} \right], \\ \delta &= \sqrt{\frac{\mu}{\mu_1 + \mu_2}} \left(e_2 \sqrt{\frac{\mu_1}{\mu_2}} - e_1 \sqrt{\frac{\mu_2}{\mu_1}} \right), \\ \rho &= \frac{\mu}{\mu_1 + \mu_2} \left(\frac{e_1 \mu_2}{\mu_1} + \frac{e_2 \mu_1}{\mu_2} \right), \\ \lambda &= \frac{\mu}{M} \left[\frac{(e_1 + e_2) \mu_3}{\mu_1 + \mu_2} + \frac{e_3 (\mu_1 + \mu_2)}{\mu_3} \right], \end{aligned} \quad (26)$$

and $e = e_1 + e_2 + e_3$ is the charge of a baryon. As a particular case, when electromagnetic fields are absent ($\Delta B = 0$) we arrive at the action derived in [3]. It follows from Eq. (24) that the center of mass coordinate R_μ is separated from relative coordinates ξ_μ and η_μ and μ plays the role of the mass of the ξ_μ, η_μ excitations. Following [3,4], the straight line approximation for strings and the asymmetric quark-diquark structure of baryons will be assumed. The asymmetric configuration (see also [9,10]) means that two quarks $q^{(1)}$ and $q^{(2)}$ are near each other and quark $q^{(3)}$ is farther from them. This case is preferable [9,10] and the slope of linear baryon Regge trajectories is the same as for mesons [3,4]. Then $|\xi| \gg |\eta|$ ($|\xi| = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$) and the coordinate η_μ can be ignored. We neglect therefore the surfaces S_1, S_2 and assume for S_3 the following expression [3,4]:

$$S_3 = b \int_0^T d\tau |\xi|, \quad (27)$$

where

$$b = \sqrt{\frac{\mu(\mu_1 + \mu_2)}{M\mu_3}} + \sqrt{\frac{\mu\mu_3}{M(\mu_1 + \mu_2)}}. \quad (28)$$

Equation (27) takes into account the confinement of quarks and gives the linear potential between quarks. Using the definition $B = \int_0^T d\tau \mathcal{L}$, where \mathcal{L} is the Lagrangian, from Eq. (25) we arrive at the effective Lagrangian for baryons

$$\mathcal{L}_{eff} = \frac{M}{2} \dot{R}_\nu^2 + \frac{\mu}{2} \dot{\xi}_\nu^2 - \sum_j \frac{m_j^2}{2\mu_j} - \sigma b |\xi| + \mathcal{L}^{el}. \quad (29)$$

Here we neglect the coordinate η_μ and introduce the notation:

$$\mathcal{L}^{el} = \frac{1}{2} F_{\nu\mu}^{el} \left[e \dot{R}_\mu R_\nu + \lambda \dot{\xi}_\mu \xi_\nu + \gamma (\dot{R}_\mu \xi_\nu + \dot{\xi}_\mu R_\nu) \right]. \quad (30)$$

Lagrangian \mathcal{L}^{el} describes the electromagnetic interaction of the string. It follows from Eq. (3.22) that

$$z_\mu^{(3)} - \frac{1}{2} (z_\mu^{(1)} + z_\mu^{(2)}) = -b\xi_\mu + \frac{1}{2} \sqrt{\frac{\mu}{\mu_1 + \mu_2}} \left(\frac{\mu_2 - \mu_1}{\sqrt{\mu_1 \mu_2}} \right) \eta_\mu. \quad (31)$$

As the second term in Eq. (31) is small, the coordinate ξ_μ is proportional to the “distance” between quark $q^{(3)}$ and the center of mass of quarks $q^{(1)}$ and $q^{(2)}$ which form a diquark. At the large time T limit $\xi_4 = 0$, $R_4 = i\tau$ [3,4] and Lagrangian (30) takes the form

$$\mathcal{L}^{el} = e(\mathbf{R}\mathbf{E}) + \gamma(\xi\mathbf{E}) - \frac{1}{2} \epsilon_{mnk} H_k (e \dot{R}_m R_n + \lambda \dot{\xi}_m \xi_n + \gamma (\dot{R}_m \xi_n + \dot{\xi}_m R_n)), \quad (32)$$

where the electric field $E_k = iF_{k4}$ and magnetic field $H_k = (1/2)\epsilon_{kmn}F_{mn}$. To clarify the physical meaning of the terms in Eq. (32), let us consider the dipole moment of quarks. Using the definition of the electric dipole moment and Eq. (22) we have

$$\mathbf{d} = \sum_j e_j \mathbf{z}^{(j)} = e\mathbf{R} + \gamma\xi + \delta\eta. \quad (33)$$

So, first two terms in Eq. (32) (neglecting the coordinate η_μ) describe the interaction of the dipole moment of quarks with the electric field in

accordance with the expression for the potential energy: $U = -(\mathbf{d}\mathbf{E})$. The magnetic moment is given by

$$m_k = \frac{1}{2}\epsilon_{mnk} \sum_j e_j z_m^{(j)} z_n^{(j)} = \frac{1}{2}\epsilon_{mnk} \left[e R_m \dot{R}_n + \lambda \xi_m \dot{\xi}_n + \gamma (R_m \dot{\xi}_n + \xi_m \dot{R}_n) \right. \\ \left. + \rho \eta_m \dot{\eta}_n + \delta (R_m \dot{\eta}_n + \eta_m \dot{R}_n) + \delta \sqrt{\frac{\mu\mu_3}{(\mu_1 + \mu_2)M}} (\xi_m \dot{\eta}_n + \eta_m \dot{\xi}_n) \right]. \quad (34)$$

It follows from Eqs. (25), (32) that there is an interaction of the magnetic field with the magnetic moment in such a way that the interaction energy is $U = -(\mathbf{m}\mathbf{H})$. So Lagrangian (32) describes the interaction of electric and magnetic moments of baryons with electric and magnetic fields, respectively.

4 Mean relative coordinate and electric polarizabilities of nucleons

Now we estimate the mean size of baryons. At the large time T limit $\dot{\xi}_0 = 0$ ($\xi_4 = i\xi_0$), $R_4 = i\tau$ [3,4] and therefore only three dimensional quantities R_k and ξ_k are important. From Eq. (32) we find three momenta corresponding to the center of mass coordinate R_k and relative coordinate ξ_k :

$$\Pi_k = \frac{\partial \mathcal{L}_{eff}}{\partial \dot{R}_k} = M \dot{R}_k - \frac{1}{2} \epsilon_{knm} H_m (e R_n + 2\gamma \xi_n), \\ \pi_k = \frac{\partial \mathcal{L}_{eff}}{\partial \dot{\xi}_k} = \mu \dot{\xi}_k - \frac{1}{2} \epsilon_{knm} H_m (\lambda \xi_n + 2\gamma R_n), \quad (35)$$

Here we take into account that Lagrangian is defined within the accuracy of the total derivative on time. Then the effective Hamiltonian $\mathcal{H}_{eff} = \Pi_k \dot{R}_k + \pi_k \dot{\xi}_k - \mathcal{L}_{eff}$ corresponding to the quark-diquark structure of a baryon is given by

$$\mathcal{H}_{eff} = \sum_j \frac{m_j^2}{2\mu_j} + \frac{M}{2} + \frac{M}{2} \dot{R}_k^2 + \frac{\mu}{2} \dot{\xi}_k^2 + \sigma b |\xi| - e(\mathbf{E}\mathbf{R}) - \gamma(\mathbf{E}\xi), \quad (36)$$

The Hamiltonian for baryons Eq. (36) looks like the one for mesons [2] because we consider basically the string between quark and diquark. Therefore the calculations of mean coordinates and electromagnetic polarizabilities

is the same. But in the case of baryons there are more parameters and the analysis is more complicated.

With the help of Eq. (35) the effective Hamiltonian (4.36) is rewritten as

$$\begin{aligned} \mathcal{H}_{eff} = & \sum_j \frac{m_j^2}{2\mu_j} + \frac{M}{2} + \frac{1}{2M} \left[\mathbf{\Pi} + \frac{e}{2} (\mathbf{R} \times \mathbf{H}) + \gamma (\xi \times \mathbf{H}) \right]^2 \\ & + \frac{1}{2\mu} \left[\pi + \frac{\lambda}{2} (\xi \times \mathbf{H}) + \gamma (\mathbf{R} \times \mathbf{H}) \right]^2 + \sigma b |\xi| - e(\mathbf{E}\mathbf{R}) - \gamma(\mathbf{E}\xi), \end{aligned} \quad (37)$$

where $(\xi \times \mathbf{H})_k = \epsilon_{mnk} \xi_m H_n$. The mass of a baryon $\mathcal{M}(\mu_j)$ is defined here as a solution to equation

$$\mathcal{H}_{eff} \Phi = \mathcal{M}(\mu_j) \Phi. \quad (38)$$

In according to the Noether theorem, the momentum $\mathbf{\Pi}$ is conserved and we can put $\mathbf{R} = \mathbf{\Pi} = 0$ in Eq. (37). To find the solution to Eq. (38) we can use the substitution $\pi_k = -i\partial/\partial\xi_k$. Then the mass of a baryon $\mathcal{M}(\mu_j)$ is given by

$$\mathcal{M}(\mu_j) = \sum_j \frac{m_j^2}{2\mu_j} + \frac{M}{2} + \epsilon(\mu_j, \mathbf{E}, \mathbf{H}), \quad (39)$$

where $\epsilon(\mu_j, \mathbf{E}, \mathbf{H})$ is the eigenvalue of the equation

$$\begin{aligned} & \left\{ \frac{1}{2\mu} \left[-i\frac{\partial}{\partial\xi} - \frac{\lambda}{2} (\xi \times \mathbf{H}) \right]^2 + \frac{\gamma^2}{2M} (\xi \times \mathbf{H})^2 + \sigma b |\xi| - \gamma(\mathbf{E}\xi) \right\} \Phi \\ & = \epsilon(\mu_j, \mathbf{E}, \mathbf{H}) \Phi. \end{aligned} \quad (40)$$

The term $(\gamma^2/(2M)) (\xi \times \mathbf{H})^2$ in Eq. (40) is due to the recoil of the string. In nonrelativistic models the effect of the recoil was studied in [13] (see also [14]). As we neglected the spin of baryons here, there is no interaction of spin with the external magnetic field. It is not difficult to take into account such interaction. Eq. (40) is like the equation for mesons [2] and therefore we write out the solution for the ground state. If $\mathbf{E} = \mathbf{H} = 0$, the solution to Eq. (40) when the orbital quantum number $l = 0$ is given by

$$\Phi(\rho, \theta, \phi) = \frac{N}{\rho} Ai(\rho - a(n)) Y_{lm}(\theta, \phi), \quad (41)$$

where $\rho = \sqrt{\rho_1^2 + \rho_2^2 + \rho_3^2}$, $\rho_k = (2\mu b\sigma)^{1/3}\xi_k$, $Ai(\rho - a(n))$ is the Airy function, $Y_{lm}(\theta, \phi)$ is the spherical function, N is the normalization constant and $a(n)$ are the Airy function zeros so that $a(1) = 2.3381$, $a(2) = 4.0879$, $a(3) = 5.52$ and so on [15]. For the ground state of baryons the principal quantum number $n = 1$. For the excited states it is necessary to choose the corresponding value of $n = n_r + l + 1$, where n_r is the radial quantum number. The eigenvalue of Eq. (40) (at $\mathbf{E} = \mathbf{H} = 0$) is

$$\epsilon(\mu_j) = (2\mu)^{-1/3} (b\sigma)^{2/3} a(n) = (\sigma)^{2/3} a(n) \left[\frac{M}{2\mu_3(\mu_1 + \mu_2)} \right]^{1/3}. \quad (42)$$

The condition of the minimum of the baryon mass (39) ($\partial\mathcal{M}(\mu_j)/\mu_j = 0$) at $m_1 = m_2$ (and $\mu_1 = \mu_2$), with the help of Eq. (41) gives the dynamical mass of a diquark:

$$\mu_3^{(0)} = \mu_1^{(0)} + \mu_2^{(0)} = \sqrt{\sigma} \left[\frac{a(n)}{3} \right]^{3/4}. \quad (43)$$

This value is different from one [3] obtained for large angular momentum. Using Eq. (43) we arrive from Eq. (39) at the expression for the mass of a baryon (see also [3]):

$$\mathcal{M}(\mu_j) = \frac{m_3^2 + 4m_1^2}{2\mu_3^{(0)}} + 4\mu_3^{(0)}. \quad (44)$$

To estimate the baryon mass, the value of the string tension $\sigma = 0.15 \text{ GeV}^2$ will be used [3,4]. Neglecting the small current masses of quarks m_j we find from Eqs. (43), (44) the mass of a diquark for $n = 1$: $\mu_3^{(0)} = 320 \text{ MeV}$ and the nucleon mass : $\mathcal{M}(\mu_j) = 1.28 \text{ GeV}$ [5]. This value of the nucleon mass is a little greater then real nucleon mass because spin-spin and spin-orbit forces were omitted.

Now we consider the mean relative coordinates of nucleons on the basis of the virial theorem which gives (as for the case of mesons [2]) the mean potential energy $\langle U \rangle = 2\langle T \rangle$, where $\langle T \rangle$ is the mean kinetic energy. Then using the relation $\langle T \rangle + \langle U \rangle = \epsilon(\mu_j)$, we arrive at

$$\langle U \rangle = \frac{2}{3}\epsilon(\mu_j) = \frac{2}{3}(2\mu)^{-1/3}(b\sigma)^{2/3}a(n). \quad (45)$$

Comparing Eq. (45) with the relation $\langle U \rangle = b\sigma\langle\sqrt{\xi^2}\rangle$ gives the following

expression

$$\langle \sqrt{\xi^2} \rangle = \frac{2}{3} (2\mu b\sigma)^{-1/3} a(n) = \sigma^{-1/4} \sqrt{\frac{2}{\mu}} \left(\frac{a(n)}{3} \right)^{9/8}. \quad (46)$$

In accordance with Eq. (31) the size of the nucleon is characterized by the value $|\mathbf{z}^{(3)} - (1/2)(\mathbf{z}^{(1)} + \mathbf{z}^{(2)})| \simeq |b\xi|$. Introducing the notation $\mathbf{r} = b\xi$, from Eq. (46) we have

$$\langle \sqrt{\mathbf{r}^2} \rangle = \frac{2}{\sqrt{\sigma}} \left(\frac{a(n)}{3} \right)^{3/4}. \quad (47)$$

The same expression was found in [1,2] for mesons. For the quark-diquark system the string tension coincides with those of mesons and therefore the quark-diquark system has approximately the same size as mesons. Using $\sigma = 0.15 \text{ GeV}^2$ and $a(1) = 2.2281$ we find the mean size of the nucleons

$$\langle \sqrt{\mathbf{r}^2} \rangle = 0.84 \text{ fm}. \quad (48)$$

The experimental value of the charge radii of the proton and neutron are $\sqrt{\langle r_p^2 \rangle} = 0.86 \text{ fm}$ [16], $\langle r_n^2 \rangle = -0.113 \pm 0.005 \text{ fm}^2$ [17]. In accordance with Eq. (4.43) the center of mass of the quark-diquark system is situated in the center between quark $q^{(3)}$ and diquark $(q^{(1)}, q^{(2)})$ and the mean radius of a nucleon is $(1/2) \langle \sqrt{\mathbf{r}^2} \rangle = 0.42 \text{ fm}$ which is the reasonable value. It should be noted that charge radii of hadrons are defined from electromagnetic formfactors.

Now let us consider the case when $\mathbf{H} = 0$, $\mathbf{E} \neq 0$. It is possible to assume as an approximation that $\mathbf{E} \parallel \xi$ [5], i.e. external electric field is parallel to the string which connects quark $q^{(3)}$ with diquark $(q^{(1)}, q^{(2)})$. So we neglect the rotation of the string. It is justified only for the ground state when the orbital quantum number $l = 0$. Introducing the effective string tension

$$\sigma_{eff} = \sigma - \frac{\gamma}{b} E, \quad (49)$$

we arrive from Eq. (40) at the eigenvalue $\epsilon(\mu_j, \mathbf{E}) = (2\mu)^{-1/3} (b\sigma_{eff})^{2/3} a(n)$. From (43), (44), by neglecting the small terms containing the current masses we find the mass of a baryon

$$\mathcal{M}(\mu_j, \mathbf{E}) = 4\sqrt{\sigma_{eff}} \left[\frac{a(n)}{3} \right]^{3/4}, \quad (50)$$

which depends on the external electric field. Inserting Eq. (49) into Eq. (50) and expanding it in a small electric field one yields

$$\mathcal{M}(\mu_j, \mathbf{E}) \simeq \left[\frac{a(n)}{3} \right]^{3/4} \left(4\sqrt{\sigma} - \frac{q}{\sqrt{\sigma}} E - \frac{q^2}{8} \sigma^{-3/2} E^2 \right), \quad (51)$$

where Eq. (43) was used and $q = e_1 + e_2 - e_3$. We write here only terms of the expansion $\mathcal{M}(\mu_j, \mathbf{E})$ in small electric field up to E^2 . The first term in Eq. (51) gives the mass of a baryon. The second one is connected with the potential energy of a dipole moment of quarks in the external electric field $U = -\mathbf{d}\mathbf{E}$. From Eq. (33) when the center of mass coordinate $\mathbf{R} = 0$ and $|\xi| \gg |\eta|$, the dipole moment of quark-diquark system is $\mathbf{d} \simeq \gamma\xi = (\gamma/b)\mathbf{r}$. Comparing the potential energy of a dipole $U = -(\gamma/b)rE$ (at $\mathbf{E} \parallel \mathbf{r}$) with the second term of Eq. (51): $-(q/\sqrt{\sigma})[a(n)/3]^{3/4}E$ we arrive at the expression for the mean relative coordinate $r = (2/\sqrt{\sigma})[a(n)/3]^{3/4}$ which coincides with Eq. (47). Here we define the electric dipole moment of quark-diquark system more precisely as compared with the letter [5] and as a result the mean relative coordinate of a baryon Eq. (47) coincides with that for a meson. The third term in Eq. (51) describes the potential energy due to the electric polarizability of a baryon. The polarization potential is given by (see [18,14])

$$U(\alpha, \beta) = -\frac{1}{2}\alpha E^2 - \frac{1}{2}\beta H^2, \quad (52)$$

where α, β are electric and magnetic static polarizabilities of hadrons, respectively. From Eq. (51) by comparing the quadratic term in \mathbf{E} with Eq. (52) we arrive at the electric polarizability of a baryon:

$$\alpha = \frac{q^2}{4} \left[\frac{a(n)}{3} \right]^{3/4} \sigma^{-3/2}. \quad (53)$$

Let us consider the estimation of the electric polarizability for the proton $p = uud$. There are two possibilities for a proton as a quark-diquark system: a) the quark $q^{(3)} = d$ and diquark $(q^{(1)}q^{(2)}) = (uu)$, so the electric charges $e_1 = e_2 = (2e)/3$, $e_3 = -e/3$ and parameter $q = e_1 + e_2 - e_3 = (5e)/3$; b) the quark $q^{(3)} = u$, diquark $(q^{(1)}q^{(2)}) = (ud)$ and the electric charges $e_1 = e_3 = (2e)/3$, $e_2 = -e/3$ and parameter $q = e_1 + e_2 - e_3 = -e/3$. It should be noted that there are no permutations of quarks here which occur in the Green function Eq. (11). Using the value of the string tension $\sigma = 0.15$

GeV² and $a(1) = 2.3381$ from Eq. (53) we find the static polarizability of a proton in Gaussian units for two cases

$$a) \alpha_p = 5.56 \times 10^{-4} \text{ fm}^3 \quad b) \alpha_p = 0.22 \times 10^{-4} \text{ fm}^3. \quad (54)$$

The generalized electric polarizability which is extracted from measurements of the Compton scattering cross sections is given by [14]

$$\bar{\alpha} = \alpha + \Delta\alpha, \quad (55)$$

$$\Delta\alpha = \frac{e^2 r_E^2}{3M_N} + \frac{e^2 (\kappa^2 + 1)}{4M_N^3}, \quad (56)$$

where M_N is the mass of a nucleon, r_E is electric radius and magnetic moment of the hadron $\mu = (e\kappa)/(2M_N)$. Using the experimental values of the electric radius and magnetic moment of a proton we have $\Delta\alpha_p = (4.5 \pm 0.1) \times 10^{-4} \text{ fm}^3$ [14]. From (54) the total electric polarizability of a proton is becomes

$$a) \bar{\alpha}_p = 10 \times 10^{-4} \text{ fm}^3 \quad b) \bar{\alpha}_p = 4.7 \times 10^{-4} \text{ fm}^3. \quad (57)$$

The configuration of a proton a), when the quark $q^{(3)} = d$ and diquark $(q^{(1)}q^{(2)}) = (uu)$ is more favorable as the experimental values of electric polarizability are

$$\bar{\alpha}_p^{\text{exp}} = (12.1 \pm 0.8 \pm 0.5) \times 10^{-4} \text{ fm}^3 \quad [19],$$

$$\bar{\alpha}_p^{\text{exp}} = (10.6 \pm 1.2 \pm 1.0) \times 10^{-4} \text{ fm}^3 \quad [20],$$

$$\bar{\alpha}_p^{\text{exp}} = (9.8 \pm 0.4 \pm 1.1) \times 10^{-4} \text{ fm}^3 \quad [21].$$

So in the case a) we have a good agreement with experimental data.

For the neutron $n = udd$ there are also two possibilities: a) the quark $q^{(3)} = u$ and diquark $(q^{(1)}q^{(2)}) = (dd)$, so the electric charges $e_1 = e_2 = -e/3$, $e_3 = (2e)/3$ and parameter $q = e_1 + e_2 - e_3 = -(4e)/3$; b) the quark $q^{(3)} = d$, diquark $(q^{(1)}q^{(2)}) = (ud)$ and the electric charges $e_3 = e_2 = -e/3$, $e_1 = (2e)/3$ and parameter $q = e_1 + e_2 - e_3 = (2e)/3$. Inserting these parameters into Eq. (53) one gives

$$a) \alpha_n = 3.56 \times 10^{-4} \text{ fm}^3 \quad b) \alpha_n = 0.89 \times 10^{-4} \text{ fm}^3. \quad (58)$$

For the neutron $\Delta\alpha_n = 0.62 \times 10^{-4} \text{ fm}^3$ [14] and the generalized electric polarizability of a neutron is given by

$$a) \bar{\alpha}_n = 4.2 \times 10^{-4} \text{ fm}^3 \quad b) \bar{\alpha}_n = 1.5 \times 10^{-4} \text{ fm}^3. \quad (59)$$

The experimental situation for a neutron is more complicated as there are different experimental data:

$$\bar{\alpha}_n^{\text{exp}} = (12.6 \pm 1.5 \pm 2.0) \times 10^{-4} \text{ fm}^3 \quad [22],$$

$$\alpha_n = (8.8 \pm 2.4 \pm 3) \times 10^{-4} \text{ fm}^3 \quad [23].$$

The recent data [23] are close to case a) in Eqs. (59) with quark $q^{(3)} = u$ and diquark $(q^{(1)}q^{(2)}) = (dd)$. The value of $\bar{\alpha}_n$ in the situation a) in Eq. (59) is close to one obtained in the oscillator nonrelativistic quark model [24-26,18,14]. The results of calculations in the framework of the dispersion sum rule [27] and the chiral perturbation theory (CHPT) [28] are closer to the case a).

5 Diamagnetic polarizabilities of nucleons

In according to Eq. (52) for calculating the magnetic polarizability of nucleons one needs to compare it with Eq. (37). The effective Hamiltonian (37) (at $\mathbf{E} = 0$, $\mathbf{R} = \mathbf{\Pi} = 0$) can be cast into

$$\begin{aligned} \mathcal{H}_{eff} &= \mathcal{H}_0 + \mathcal{H}_{int}, \\ \mathcal{H}_0 &= \sum_j \frac{m_j^2}{2\mu_j} + \frac{M}{2} - \frac{1}{2\mu} \frac{\partial^2}{\partial \xi_j^2} + \sigma b |\xi|, \\ \mathcal{H}_{int} &= -\frac{\lambda}{2\mu} \mathbf{H} \mathbf{L} + \left(\frac{\lambda^2}{8\mu} + \frac{\gamma^2}{2M} \right) [(\xi \times \mathbf{H})]^2, \end{aligned} \quad (60)$$

where $L_k = -i\epsilon_{kmn}\xi_m\partial_n$ is the angular momentum and $\partial_n = \partial/\partial\xi_n$. Without loss of generality we can choose the direction of the magnetic field on the third axis, i.e. $\mathbf{H} = (0, 0, H)$. Then the Hamiltonian of an interaction of quarks with the magnetic field is given by

$$\mathcal{H}_{int} = -\frac{\lambda H}{2\mu} L_3 + H^2 \left(\frac{\lambda^2}{8\mu} + \frac{\gamma^2}{2M} \right) (\xi_1^2 + \xi_2^2), \quad (61)$$

where $L_3 = i(\xi_2\partial_1 - \xi_1\partial_2)$. Considering the small external magnetic field, the perturbative theory can be applied. Using the perturbative method [28] one arrives at the shift of the energy

$$\Delta\mathcal{E}_n = \langle n | \left[-\frac{\lambda H}{2\mu} L_3 + \left(\frac{\lambda^2}{8\mu} + \frac{\gamma^2}{2M} \right) H^2 \xi^2 \sin^2 \vartheta \right] | n \rangle$$

$$+ \sum_{n'} \frac{|\langle n' | -(\lambda H L_3) / (2\mu) | n \rangle|^2}{\mathcal{E}_n - \mathcal{E}_{n'}}, \quad (62)$$

where ϑ is the angle between coordinate ξ and magnetic field \mathbf{H} . If the first term in Eq. (62) is not equal to zero then the second and third terms are smaller, and the main contribution to the energy comes from the first term. This occurs when the orbital quantum number $l > 0$. In the case $l = 0$, the shift of the energy due to the interaction with the magnetic field is defined by the second term in Eq. (62).

After averaging Eq. (62) and taking into account the equation $(1/4\pi) \int \sin^2 \vartheta d\Omega = 2/3$ we find for the ground state when $l = 0$, $L_3 | 0 \rangle = 0$, the shift of energy

$$\Delta \mathcal{E}_n = \left(\frac{\lambda^2}{4\mu} + \frac{\gamma^2}{M} \right) \frac{H^2}{3} \langle \xi^2 \rangle, \quad (63)$$

where $\langle \xi^2 \rangle = \langle 0 | \xi^2 | 0 \rangle$, and $| 0 \rangle$ means the wave function of the ground s -state. We took into account that the first and the third terms in Eq. (62) equal zero because $L_3 | 0 \rangle = 0$. Here we ignore the spin interactions of a baryon with the external magnetic field and therefore only diamagnetic polarizability can be defined from Eq. (63). Using the definition of the relative coordinate $\mathbf{r} = b\xi$ and comparing Eq. (63) with Eq. (52) gives the diamagnetic polarizability of a baryon

$$\beta^{dia} = - \left(\frac{\lambda^2}{4\mu} + \frac{\gamma^2}{M} \right) \frac{2}{3b^2} \langle \mathbf{r}^2 \rangle. \quad (64)$$

As required, the diamagnetic polarizability is negative and Eq. (64) is like the Langevin formula for the magnetic susceptibility of atoms. The similar expression was derived in [2] for mesons. Take into account Eqs. (26), (43), expression (64) is rewritten as

$$\beta^{dia} = - \left(\frac{e^2}{4} + q^2 \right) \frac{\langle \mathbf{r}^2 \rangle}{6M}, \quad (65)$$

where $M = \mu_1^{(0)} + \mu_2^{(0)} + \mu_3^{(0)} = 2\mu_3^{(0)} = 2\sqrt{\sigma} [a(n)/3]^{3/4}$. To calculate the value of β^{dia} for nucleons we can use the theoretical magnitude of the mean-squared relative coordinate $\langle \mathbf{r}^2 \rangle$ or experimental data for the size of a nucleon. The first way is preferable. Taking into account Eq. (47), the value of M and using the approximate relation $\langle \mathbf{r}^2 \rangle \simeq \langle \sqrt{\mathbf{r}^2} \rangle^2$, Eq. (65) transforms into

$$\beta^{dia} = - \frac{e^2 + 4q^2}{12\sigma^{3/2}} \left[\frac{a(n)}{3} \right]^{3/4}. \quad (66)$$

For a proton with the more favorable configuration a), when the quark $q^{(3)} = d$ and diquark $(q^{(1)}q^{(2)}) = (uu)$, parameter $q = (5e)/3$ and Eq. (5.66) (at $\sigma = 0.15 \text{ GeV}^2$) takes the value

$$\beta_p^{dia} = -8 \times 10^{-4} \text{ fm}^3. \quad (67)$$

This quantity is greater than that found in the nonrelativistic quark model [18,14]. The value of the magnetic polarizability extracted from the low-energy Compton experiments [19-22] is

$$\bar{\beta} = \beta^{para} + \beta^{dia}, \quad (68)$$

where the paramagnetic polarizability is given by [18,14]

$$\beta^{para} = 2 \sum_n \frac{|\langle n | m_z | 0 \rangle|^2}{\mathcal{E}_n - \mathcal{E}_0}. \quad (69)$$

Here $|0\rangle$, $|n\rangle$ are the ground and excited states of a nucleon, respectively, m_z is the third projection of the magnetic dipole operator. The main contribution to the paramagnetic polarizability of a nucleon is due to from $\Delta(1232)$ excitation [28] and is given by $\beta_{\Delta}^{para} = (13 \pm 3) \times 10^{-4} \text{ fm}^3$. The intermediate state Δ is an isovector excitation and contributes to the proton and neutron so that $\beta_{\Delta}^p = \beta_{\Delta}^n$. To calculate β^{para} in the present approach one needs to take into account the interaction of spins of quarks with the magnetic field. For estimation of the total magnetic polarizability of a proton we use the contribution β_{Δ}^{para} and in accordance with Eqs. (67), (68) we find

$$\bar{\beta}_p = (5 \pm 3) \times 10^{-4} \text{ fm}^3. \quad (70)$$

This quantity is close (within two standard deviations) to the experimental value [22] which was extracted from the measurements of the cross sections of the Compton scattering on hydrogen: $\bar{\beta}_p^{\text{exp}} = (2.9 \mp 0.7 \mp 0.8) \times 10^{-4} \text{ fm}^3$. The value (70) is also in agreement with $\bar{\beta}_p^{\text{exp}} = (2.1 \mp 0.8 \mp 0.5) \times 10^{-4} \text{ fm}^3$ [19] and with the quantity found in CHPT [28]. The value $\bar{\beta}_p$ is small due to the partial cancellation of the positive paramagnetic polarizability $\bar{\beta}_p^{para}$ with the negative diamagnetic polarizability $\bar{\beta}_p^{dia}$. The theoretical evaluation, interpretation and experimental extraction of the total magnetic polarizability $\bar{\beta}_p$ is difficult because it is small. That is why this quantity is model dependent in different schemes. The approach considered allows us to improve the

accuracy by using the perturbation in the spin interaction. The next step is to take into account such corrections.

For a neutron with the configuration a) where the quark $q^{(3)} = u$ and diquark $(q^{(1)}q^{(2)}) = (dd)$, the parameter $q = -(4e)/3$ and Eq. (65) gives

$$\beta_n^{dia} = -5.4 \times 10^{-4} fm^3. \quad (71)$$

Using the paramagnetic polarizability $\beta_\Delta^{para} = (13 \pm 3) \times 10^{-4} fm^3$ [30] for a neutron we get from (68) the generalized magnetic polarizability of a neutron

$$\bar{\beta}_n = (7.6 \pm 3) \times 10^{-4} fm^3 \quad (72)$$

which is in agreement with the experimental quantities

$$\beta_n = (6.5 \mp 2.4 \mp 3) \times 10^{-4} fm^3 \quad [23]$$

$$\beta_n^{exp} = (3.2 \mp 1.5 \mp 2.0) \times 10^{-4} fm^3 \quad [22]$$

According to Eqs. (57), (59), (70), (72) the sum of nucleon polarizabilities in our approach is given by

$$\bar{\alpha}_p + \bar{\beta}_p = (15 \pm 3) \times 10^{-4} fm^3,$$

$$\bar{\alpha}_n + \bar{\beta}_n = (11.8 \pm 3) \times 10^{-4} fm^3. \quad (73)$$

Values (73) are close to the reliable results found on the basis of the sum rule [27]. The interesting feature of the experimental data is that for the proton and the neutron the approximate equality $\bar{\alpha}_p + \bar{\beta}_p \simeq \bar{\alpha}_n + \bar{\beta}_n$, and inequality $\bar{\alpha}_p, \bar{\alpha}_n \gg \bar{\beta}_p, \bar{\beta}_n$ are valid. The last inequality means that both a proton and a neutron behave like electric dipoles. In addition, the large positive paramagnetic polarizability of nucleons should be cancelled by the diamagnetic contribution. It was shown in the framework of the sum rule [27] that in the chiral limit, $m_\pi \rightarrow 0$, both polarizabilities $\bar{\alpha}$, $\bar{\beta}$ diverge as $O(m_\pi^{-1})$, and this was confirmed in CHPT [28]. The same behavior of pion polarizabilities was found on the basis of quasiclassical calculations [31] in the framework of the instanton vacuum theory. Thus, the leading terms of electromagnetic polarizabilities are determined by CSB. As pion degrees of freedom play a very important role in the nucleon electric polarizability, one needs to describe pions in this scheme. In the approach considered here the accuracy of the values of electromagnetic polarizabilities can be improved by taking into consideration pion degrees of freedom.

6 Conclusion

The model of baryons as a quark-diquark system is very similar to the approach for mesons [2]. The effective action for baryons (21) obtained on the basis of the QCD string theory is applied here for calculating the mean size and electromagnetic polarizabilities of a proton and neutron which are in reasonable agreement with the experimental data. The preferable combination for a diquark is (uu) for a proton and (dd) for a neutron. In this case theoretical values for electromagnetic polarizabilities are close to experimental data.

It should be noted that the quark-diquark ansatz used here generates an electric dipole moment, and as a result, there is a contribution to the energy linear in the electric field which is like the linear Stark effect.

As a first step, we made some approximations and model assumptions. So, spin interactions of quarks treated as a perturbation were neglected here, but we took them into account by using the paramagnetic polarizability of a nucleon due to the Δ contribution. As an approximation, it is justified, because in Isgur-Karl model of baryons [32] spin-orbit splitting are much smaller than expected from one-gluon-exchange matrix elements (spin forces were also discussed in [33]). It was also shown [34,35] that the contributions from the Coulomb and spin-spin interactions cancel each other. Nevertheless all parameters should be taken from the approach and then compared with empirical data. Therefore one needs to calculate the paramagnetic polarizability of a nucleon in our approach.

Implying small spin-orbit forces in baryons, we come to K^* and quark-diquark system correspondence (see [2]). It is possible also to consider excited states of baryons at nonzero orbital, l , and principal, n , quantum numbers. So, the present approach can be applied for studying any baryons (see also [34]).

The electromagnetic polarizabilities of nucleons found are close to the values calculated in the framework of the dispersion sum rule [27] and CHPT [28].

To improve the accuracy of calculations of electromagnetic polarizabilities of nucleons, one should take into account pion degrees of freedom because the pion cloud contributes substantially to electromagnetic properties.

The nucleon polarizabilities were also evaluated on the basis of nucleon soliton models in [36-42] .

References

- [1] S. I. Kruglov, Phys. Lett. B **390**, 283 (1997).
- [2] S. I. Kruglov, Phys. Rev. D **60**, 116009 (1999).
- [3] Yu. A. Simonov, Phys. Lett. B **226**, 151 (1989); B **228**, 413 (1989).
- [4] Yu. A. Simonov, Nucl. Phys. B **307**, 513 (1988); B **324**, 67 (1989);
Yad. Fiz. **54** 192 (1991).
- [5] S. I. Kruglov, Phys. Lett. B **397**, 283 (1997).
- [6] H. G. Dosch, Phys. Lett. B **190**, 177 (1987); H. G. Dosch, Yu. A. Simonov, Phys. Lett. B **205**, 399 (1988).
- [7] G. 't Hooft, in High Energy Physics (Ed. A. Zichichi) (Bologna: Editrice Compositori, 1976); S. Mandelstam, Phys. Lett. B **53**, 476 (1975).
- [8] G. 't Hooft, Nucl. Phys. B **190**, 455 (1981).
- [9] S. Fleck, B. Silvestre-Brac, and J. M. Richard, Phys. Rev. D **38**, 1519 (1988).
- [10] J.-L. Basdevant and Boukraa, Z. Phys. C **30**, 103 (1986); A. Martin, Z. Phys. C **32**, 359 (1986).
- [11] L. Faddeev, V. Popov, Phys. Lett. B **25**, 29 (1969).
- [12] M. Campostrini, A. Di Giacomo, G. Mussardo, Z. Phys. C **25**, 173 (1984); A. Di Giacomo and H. Panagopoulos, Phys. Lett. B **285**, 133 (1992).
- [13] J. L. Friar, Ann. Phys. (N.Y.) **95**, 170 (1975).
- [14] A. I. L'vov, Int. J. Mod. Phys. A **8**, 5267 (1993).
- [15] Handbook of Mathematical Functions, edited by M. Abramowitz and I. A. Stegun, National Bureau of Standards Applied Mathematics Series, No. 55 (U.S.G.P.O., Washington, D. C., 1964).
- [16] G. G. Simon et al., Nucl. Phys. A **333**, 381 (1980).

- [17] S. Kopecky et al., Phys. Rev. Lett. **74**, 2427 (1995).
- [18] V. A. Petrun'kin, Sov. J. Part. Nucl. **12**, 278 (1981).
- [19] B. E. MacGibbon et al., Phys. Rev. **C52**, 2097 (1995); preprint nucl-ex/9507001.
- [20] A. Zieger et al., Phys. Lett. B **278**, 34 (1992).
- [21] E. L. Hallin et al., Phys. Rev. C **48**, 1497 (1993).
- [22] J. Schmiedmayer et al., Phys. Rev. Lett. **66**, 1015 (1991).
- [23] M. Lundin et al., Phys. Rev. Lett. **90**, 192501 (2003).
- [24] S. Ragusa, Lett. Nuovo Cim. **1**, 416 (1971).
- [25] G. Dattoli, G. Matone and D. Prosperi, Lett. Nuovo Cim. **19**, 601 (1977).
- [26] B. R. Holstein, Comments on Nucl. Part. Phys. **20**, 301 (1992).
- [27] A. I. L'vov, Phys. Lett. B **304**, 29 (1993).
- [28] V. Bernard, N. Kaiser and Ulf-G. Meissner, Phys. Rev. Lett. **67**, 1515 (1991); Nucl. Phys. B **373**, 346 (1992); V. Bernard, N. Kaiser, A. Schmidt and Ulf-G. Meissner, Phys. Lett. B **319**, 269 (1993).
- [29] S. Flügge, Practical Quantum Mechanics I (Springer-Verlag, Berlin, 1971).
- [30] N. C. Mukhopadhyay, A. M. Nathan, L. Zhang, Phys. Rev. D **47**, R7 (1993).
- [31] S. I. Kruglov, J. Phys. G **22**, 461 (1996).
- [32] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978).
- [33] D. Gromes and I. O. Stamatescu, Nucl. Phys. B **112**, 213 (1976); D. Gromes, ibid. B **130**, 18 (1977); Z. Phys. C **26**, 401 (1984).
- [34] M. Fabre de la Rippe and Yu. A. Simonov, Ann. Phys. **212**, 235 (1991).
- [35] Yu. A. Simonov, hep-ph/9911237.

- [36] E. M. Nyman, Phys. Lett. B **142**, 388 (1984).
- [37] M. Chemtob, Nucl. Phys. A **473**, 613 (1987).
- [38] N. N. Scoccola and W. Weise, Phys. Lett. B **232**, 287 (1989); Nucl. Phys. A **517**, 495 (1990).
- [39] W. Broniowski, M. K. Banerjee and T. D. Cohen, Phys. Lett. B **283**, 22 (1992); W. Broniowski and T. D. Cohen, Phys. Rev. D **47**, 299 (1993);
- [40] S. Scherer and P. J. Mulders, Nucl. Phys. A **549**, 521 (1992).
- [41] B. Schwesinger, Phys. Lett. B **298**, 17 (1992).
- [42] S. I. Kruglov and M. V. Polyakov, “Rapid communications on theoretical physics”, preprint 652(2) (Stepanov Institute of Physics, Minsk, 1992), p. 31.